The Winning Strategy for the Sparse Ruler Game.

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BACKGROUND:

- The Sparse Ruler is an impartial combinatorial game meaning the available moves depend only on the position (ruler size), not on which player is moving
- The goal of the study was to analyze optimal winning strategy for this game
- The Sprague-Grundy Theorem was used to classify every possible position a player might have as either P-positions (losing) or N-position (winning).

METHODS

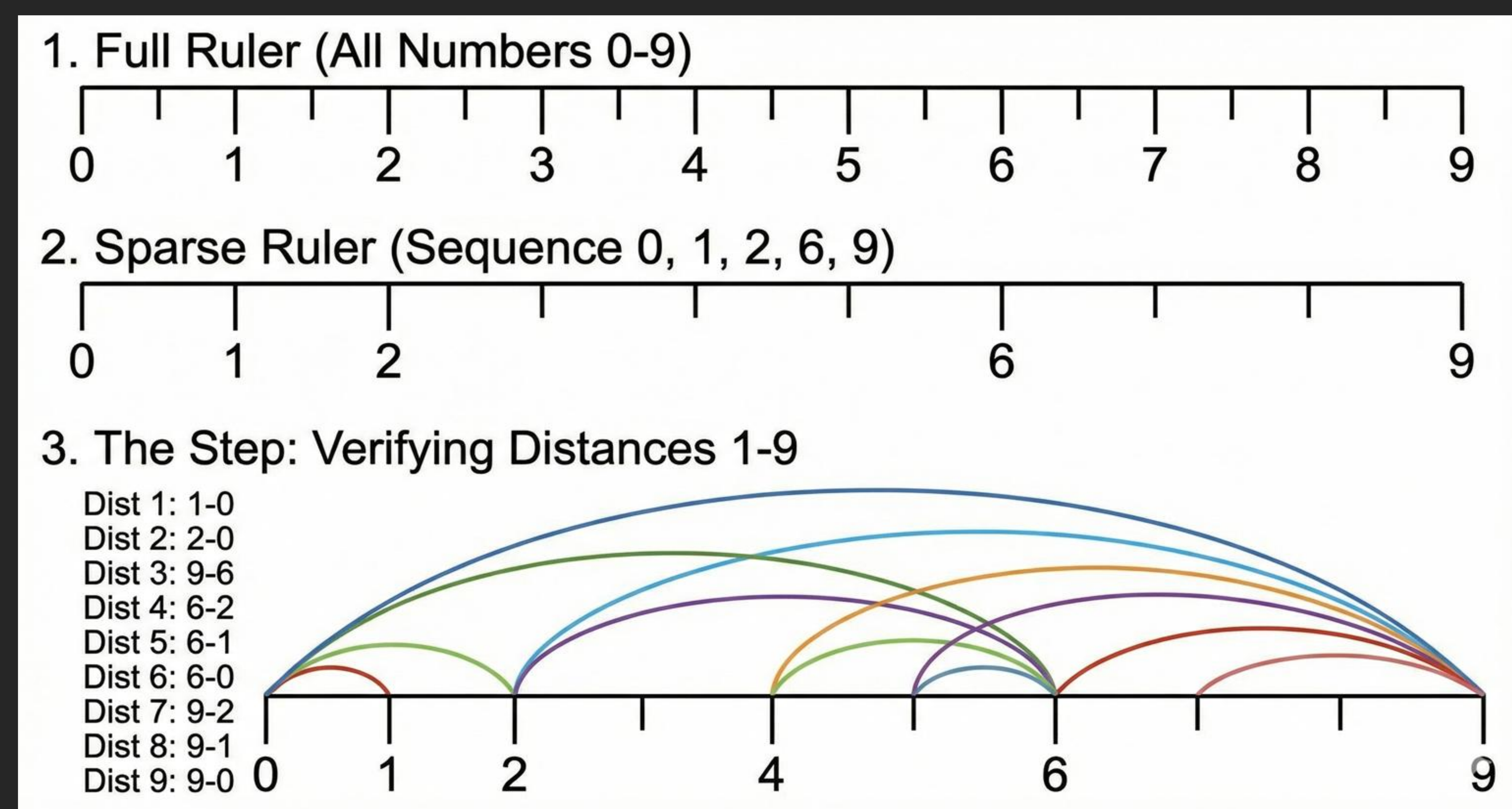
1. The Sparse Ruler is played by alternately adding integers marks to a ruler, so all distances stay measurable. The player who makes the last move wins.
2. A custom Python script was used to evaluate every possible game state of a ruler sequence and determine its starting P/N position.
3. The program classified a state as N if it has at least one legal move that leads to a P-position
4. Otherwise, it was classified as P if all legal moves from the current position led to N-positions
5. This classification was confirmed and extended by computing the Grundy value $\mathcal{G}(n)$ using the MEX rule for ruler sizes up to 20, where $\mathcal{G}(n) = 0$ is the P-position

RESULTS

- For ruler sizes $n = 1$ to $n = 7$, the Grundy values follow an irregular pattern within {0, 1, 2}.
- For sizes $n \geq 8$, the values settle into a stable alternation: odd n yield 0 (P-positions) and even n yield 1 (N-positions).

Ruler Size (n)	Grundy Number $\mathcal{G}(n)$	Position Type	Guaranteed Winner
Odd	0	P-position	Second Player
Even	1	N-position	First Player

Starting at ruler size 9, odd **rulers** always favor the **second player**, while **even rulers** always favor the **first player**, under perfect play.



Take a picture to **download** the **full GitHub repository**

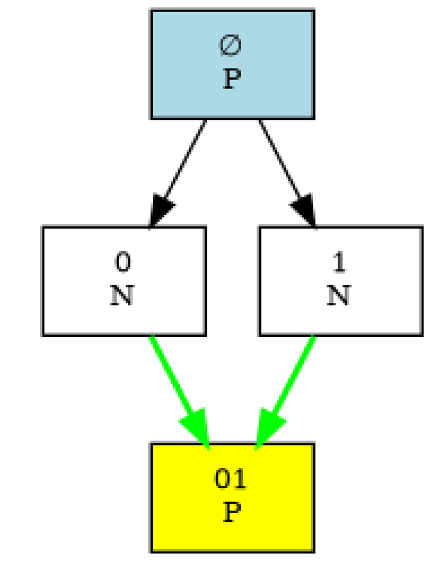


Figure 1: Digitally generated game tree of sparse ruler with size 1.

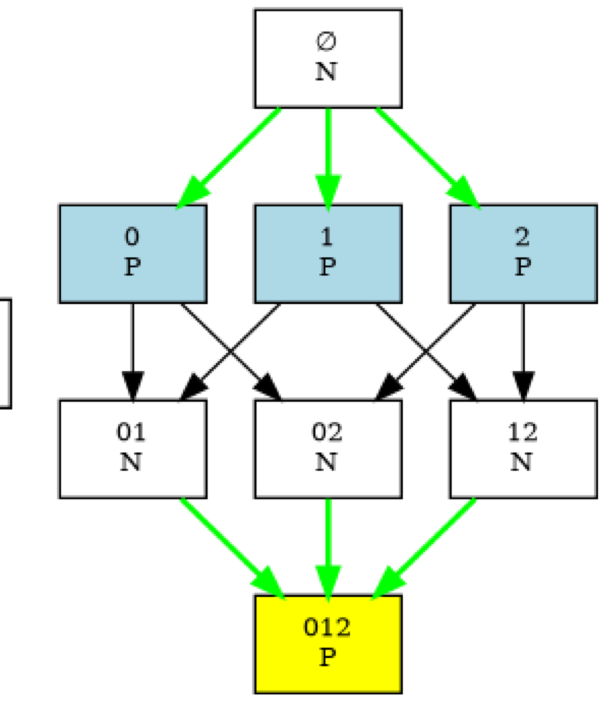


Figure 2: Digitally game tree of sparse ruler with size 2

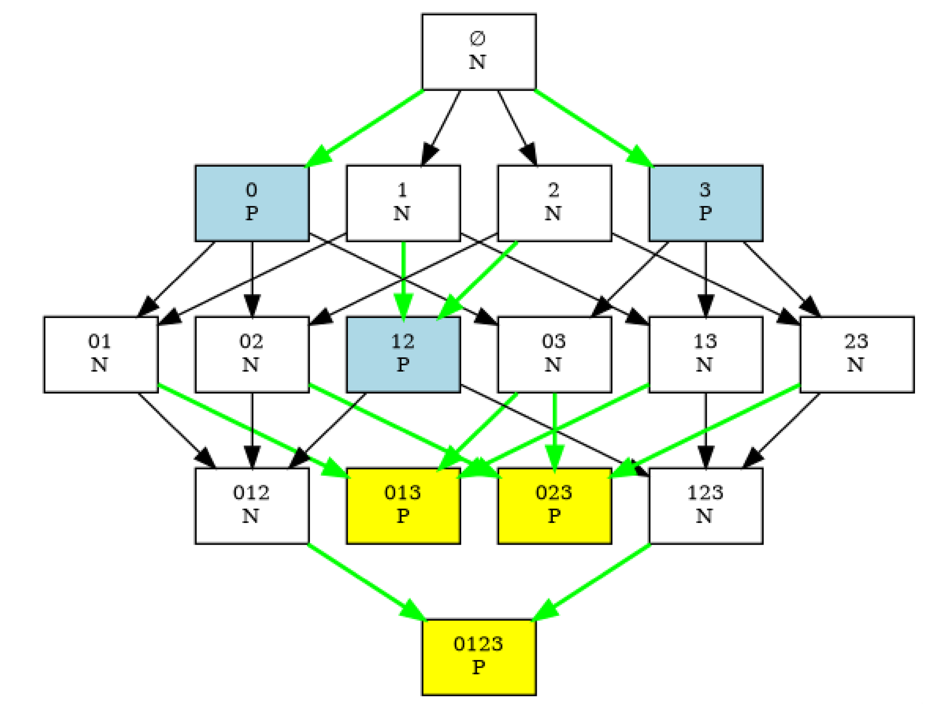


Figure 3: Digitally generated game tree of sparse ruler with size 3

Represent the terminal p-position of the game.
Blue represents the normal p-position.

Grundy Number General Equation

$$G(P) = \text{mex}\{\, G(Q) \mid Q \in \text{Options}(P) \,\}$$

where

$$\text{mex}(S) = \text{the smallest non-negative integer not in } S$$

n	1	2	3	4	5	6	7	8	9	10
$\mathcal{G}$(n)	0	1	2	0	0	1	2	1	0	1
n	11	12	13	14	15	16	17	18	19	20
$\mathcal{G}$(n)	0	1	0	1	0	1	0	1	0	1

Table 1: Computed Grundy number value for ruler size.

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